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$$\therefore\ \$4.\frac{\dot{2}9\dot{7}}{1.00\dot{3}}=\$4\frac{330}{11137}=\$\frac{44878}{11137}=\text{cost price.}$$

$$\$6.\frac{1000}{33337}=\$6\frac{100}{33337}=\$\frac{200122}{33337}=\text{selling price.}$$

$$\therefore\ \$\frac{200122}{33337}-\$\frac{44878}{11137}=\$\frac{19801644}{100344437}=\text{gain.}$$

$$\left(\frac{19801644}{100344437}\div\frac{44878}{11137}\right)\text{ of }100\%=48\frac{19640328}{20217539}\%.$$

Also solved by *O. S. WESTCOTT* and *ALOIS F. KOVARIK.*

113. Proposed by B. F. SINE, Principal of Normal School, Capon Bridge, W. Va.

In what time can a note of \$5280, bearing 6% interest, be paid by paying \$600 a year? [Solve by arithmetic].

Solution by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.

Let Q be the principal, r the rate, n the number of years, and P annual payment. Then

$Q(1+r)$=amount due at end of first year.

$Q(1+r)-P$=principal to run second year.

$Q(1+r)^2-P(1+r)$=amount due at end of second year.

$Q(1+r)^2-P(1+r)-P$=principal to run third year.

....

$Q(1+r)^n-P(1+r)^{n-1}\ldots\ldots P(1+r)-P$=amount to run $(n+1)$th year.

But the debt is cancelled. Hence

$$Q(1+r)^n-P(1+r)^{n-1}-\ldots\ldots P(1+r)-P=0.$$

$$\therefore P\left[\frac{(1+r)^n-1}{r}\right]=Q(1+r)^n.$$

$$\therefore\ (P-Qr)(1+r)^n=P.$$

$$\therefore\ n=\frac{\log P-\log(P-Qr)}{\log(1+r)}.$$

In the problem $P=\$600$, $Q=\$5280$, $r=.06$.

$$\therefore\ n=\frac{\log 600-\log(600-5280\times.06)}{\log(1.06)}=12.88\text{ years.}$$

Also solved by *G. B. M. ZERR*, *COOPER D. SCHMITT*, and *J. SCHEFFER.*

ALGEBRA.

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93. Proposed by CHARLES C. CROSS, Whaleyville, Va.

Given $x^x+y^y=285$, and $y^x-x^y=14$, to find the values of x and y. [From *Bonnycastle's Algebra*, 1841].

I. Solution by A. H. BELL, Hillsboro, Ill.

The two equations give

$$(285-y^y)^y=(y^x-14)^x\ldots\ldots(1).$$

In (1), assuming y equal to 3, 2.9, and 2.8, by logarithms, $y=2.8248$ nearly, $=\log 0.450987$.

Multiplying by y, $\log(y^y)=\log 1.273952, =18.791=y^y$.

But $x^x=285-y^y$.

$\therefore 285-18.791=266.791, =x^x$.

In a table of logarithms by $\log x+\log$. of $\log x$, x is found to be nearly 4.01637+.

II. Solution by J. M. BOORMAN, Consultative Mechanician, Etc., Woodmere, N. Y.

$x=4.016631(8\pm$; $x_1=4.04444(4\pm$; $x_2=4.045\pm$;* $x_a<0=-?$;

$y=2.8157(12\pm$; $y_1=-1.922(77\pm$; $y_2=-1.92\pm$; $y_a>0=+?$;

curiously enough always a quartic (even if $x=6$ or more, $y=5$ or more).

*Because $x=v$ nearly in $v^v=285$, as if $y<0$. $\therefore y^y=(1/y)^n$; $x^y=(1/x)^n=$ fractions. $\therefore$ as $v=4.0449$ nearly, x_1 (must) <4.0449, x_2 (must) >4.0449, so $y_2^{x_2}$ (must)$=14.069$ nearly by (2), as $x_2^{y_2}=$a negative fraction$=0.069\pm$. Solve $10^{x\log x}+10^{y\log y}=10^{\log 285}$......(1), and $10^{x\log y}-10^{y\log x}=10^{\log 14}$ .....(2), by double position. Note that $10^x=10^{4016631}=4016631$ power,—four millionth, etc., power of the millionth root of 10. So, curiously, we must compute to an even decimal if $x>0$, and to an odd decimal digit if $x<0$, to get true results.

III. Solution by OLIVER S. WESTCOTT, Principal North Division High School, Chicago, Ill.

I know of no direct method of solution, and as *Bonnycastle's Algebra*, from which it was taken, gives some idea of the use of logarithms and also of the rule known to arithmeticians in the days of Daboll as Double Position, it is presumable at least, that Bonnycastle's method was something like the following:

If x were 4 and y were 3, we should have $4^4+3^3=283$ and $3^4-4^3=17$.

Evidently then, $x>4$ and $y<3$.

Put $y=2.75$. Then $x^x+2.75^y=285$, and $x^x=268.85028$.

$x\log x=\log 268.85028$, $x=4.0205$.

Substituting these values of x and y in equation (2) we have

$$(2.75)^{4.0205}-(4.0205)^{2.75}=12.494296. \quad 14-12.494296=1.50570 \text{ error}.$$

Again, put $y=2.85$. Then $x^x+(2.85)^{2.85}=285$. $x^x=265.21628$. $x\log x=\log 265.21628$. $x=4.0148$.

Substituting as before,

$$(2.85)^{4.0148}-(4.0148)^{2.85}=14.471434. \quad 14.471434-14=0.471434 \text{ error},$$

$$\text{and } \frac{0.471434\times2.75+1.505704\times2.85}{1.505704+0.471434}=2.82615$$

for an approximate value of y.

Since 2.85 is plainly much nearer the result sought for than 2.75, put $y=2.83$.

Then $x^x+(2.83)^{2.83}=285$. $x^x=266.00867$. $x\log x=\log 266.00867$. $x=4.01605$.

Substituting in equation (2) as before $(2.83)^{4.01605}-(4.01605)^{2.83}=14.083487$. $14.083487-14=0.083487$ error. Then

$$\frac{0.471434\times2.83-0.083487\times2.85}{0.471438-0.083487}=2.825696.$$

Bonnycastle's answer for y is 2.825716.

Again $x^x+(2.825696)^{2.285696}=285$. $x^x=266.17636274$. $x\log x=\log 266.17636274$. $x=4\ 016348$.

Bonnycastle's answer for x is 4.016698.

These trifling discrepancies may easily be occasioned by variation in the logarithmic tables used. I have made use of Chambers's seven place tables.

I have not succeeded in effecting a neat and satisfactory solution; by trial, however, I find, approximately, $x=3.89$, $y=3.54$. G. B. M. ZERR.

I do not believe a strictly algebraic solution of this problem can be given, because it is not an algebraic, but a transcendental function. Algebraically there are really four unknowns, x, $\log x$, y, and $\log y$. Of course, $\log x$ and $\log y$ are known if x and y are known, but they are here algebraically separate unknowns because of the manner in which they enter the equations. W. H. DRANE.

GEOMETRY.

114. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

If a variable ellipse hyperosculate a fixed ellipse at the extremity of the minor axis, the locus of the foci is a circle whose diameter is equal to the radius of curvature.

Solution by the PROPOSER.

Let $x^2/a^2+y^2/b^2=1$......(1) be the given ellipse; then the tangent at the vertex of the minor axis is $y/b-1=0$......(2), and the variable conic is

$$x^2/a^2+y^2/b^2-1-\lambda[(y/b)-1]^2=0......(3), \text{ or}$$

$$x^2/a^2+(1-\lambda)(y^2/b^2)+(2\lambda/b)y-(\lambda+1)=0\ .....(4).$$

Now, the foci of the general conic

$$ax^2+2hxy+by^2+2gx+2fy+c=0......(5),$$

are given by

$$C(x^2-y^2)+2Fy-2Gx+A-B=0......(6),$$